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Bound States of D(2p)-D0 Systems and Supersymmetric p-Cycles

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Abstract

We discuss some issues related to D(2p)-D0 branes with background magnetic fluxes respectively, in a T-dual picture, Dp-Dp branes at angles. In particular, we describe the nature of the supersymmetric bound states appearing after tachyon condensation. We present a very elementary derivation of the conditions to be satisfied by such general supersymmetric gauge configurations, which are simply related by T-duality to the conditions for supersymmetric p-cycles in $\mathbb{C}^p$.

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1. Introduction

Recently, in [1,2] configurations of D($2p$)-D0 branes with background fluxes were discussed. In particular, in [2] the BPS bound state of a D6-D0 system in a non-vanishing constant background B-field was studied using the effective field theory on the D0-brane. Analyzing the preserved supercharges for such a configuration, it was found that for a certain codimension one sublocus the system becomes supersymmetric. On one side of the supersymmetry locus the system is in a stable non-supersymmetric configuration whereas it was argued that on the other side to decay into a stable $\frac{1}{8}$BPS bound state.

As was discussed in [3], by a certain T-duality these configurations are mapped to D-branes at angles [4], for which the supersymmetry conditions are well known. Recently, such general configurations of D-branes at angles were used in [5,6,7,8] to construct non-supersymmetric open string vacua with some appealing phenomenological properties like chirality, supersymmetry breaking, three generations in standard model like gauge theories, hierarchy of Yukawa couplings and supression of proton decay. In this context the question arose, what kind of decay is triggered by the open string tachyons localized at the intersections of D-branes. The result of [2] indicates that via a Higgs mechanism in the effective theory two tachyonic intersecting D-branes will decay into a supersymmetric configuration.

In this letter, we analyze the nature of these bound states for the D4-D0, D6-D0 and D8-D0 systems in some more detail. In particular, in section 2 we review the effects of a toroidal compactification on such D-brane configurations. Section 3 is devoted to an analysis of the decays of the D($2p$)-D0 systems. We argue that the presence of a tachyon signals the existence of a supersymmetric configuration in the same topological sector, but with lower energy than the D($2p$)-D0 system. Concerning both the number of preserved supersymmetries and the broken gauge group such a decay is in agreement with the proposed Higgs mechanism [2] in the effective field theory description. Note, that the analogous transition has been studied in the Calabi-Yau setting in [9].

The equations describing such $2p$-dimensional supersymmetric gauge configurations are given by a T-dual version of the conditions for supersymmetric p-cycles and generalize the self-duality constraint for BPS gauge configurations in four dimensions. We will derive these equations in a very elementary way by lifting the global supersymmetry conditions, $\sum_j \Phi_j = \pi$, to local ones. This allows us to straightforwardly derive the supersymmetry conditions, which indeed turn out to be related by T-duality to the conditions of supersymmetric p-cycles [10] in $\mathbb{C}^p$.

2. D($2p$)-D0 branes on a torus

In this section we first review the connection of D($2p$)-D0 branes with background fluxes and Dp-Dp branes intersecting at angles. We complexify the transversal directions of the D0-brane inside the D($2p$)-brane

$$z_j = x_j + i\, y_j \tag{2.1}$$

with $j \in \{1, \ldots, p\}$. Note that the "real-part directions" are given by the Dp-brane that was the D0-brane before the T-dualities were performed. In the following we will trade a background B-flux for a background magnetic flux on the D-branes. A constant magnetic flux, F, on the D($2p$) brane of the form

$$F = \bigoplus_{j=1}^{p} \begin{pmatrix} 0 & F_{(2j-1),2j} \\ -F_{(2j-1),2j} & 0 \end{pmatrix} \tag{2.2}$$

is mapped via T-duality along all p directions x_{2j-1} into Dp-Dp branes at angles as shown for the case of D3-branes in figure 1

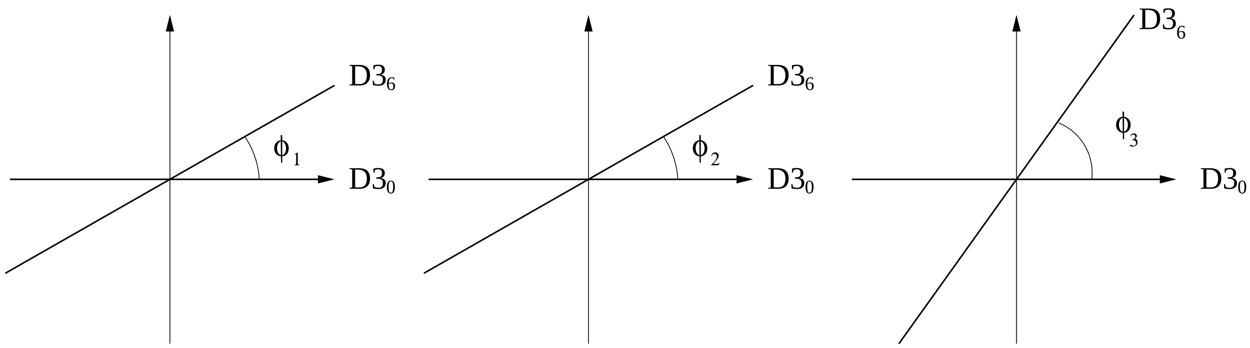

Fig.1: Branes at angles

The p angles are related to the magnetic fluxes by

$$F_{(2j-1),2j} = \cot \Phi_j . \tag{2.3}$$

Note, that the relation between the angles Φ_j and the phases v_j in [2] is $\Phi_j = \pi(1/2 - v_j)$. First compactify the coordinates (2.1) on a torus T^{2p} leading to Dp-Dp branes at angles. In the case of T-dual D(9-p)-D(9-p) branes filling all non-compact directions, one has to satisfy certain non-trivial RR tadpole cancellation conditions. In the following let us review these conditions. In order to write them down, we observe that in the branes at

angles picture a D1-brane of finite length on a two-dimensional torus T^2_j is described by two wrapping numbers (n_j, m_j) along the two fundamental cycles $[\mathbf{2j-1}]$ and $[\mathbf{2j}]$ of the torus. The angle of this D1-brane with the x-axis is given by

$$\cot\Phi_j = \frac{n_j R^{(2)}_j}{m_j R^{(1)}_j}, \tag{2.4}$$

where $R^{(1)}_j$ and $R^{(2)}_j$ denote the two radii of each T^2_j. Under T-duality (2.4) is mapped to the following discrete values of the magnetic flux

$$F_{(2j-1),2j} = \frac{n_j}{m_j R^{(1)}_j R^{(2)}_j}, \tag{2.5}$$

so that n_j can be interpreted as the magnetic charge, $c_1(F)$, of the gauge bundle and m_j as the winding number of the $D2$-brane around the torus T^2_j. Say, that we have K different stacks of $N^{(i)}$ D-branes with $i \in \{1, \ldots, K\}$ and that the D-branes are wrapped exactly around one 1-cycle on each two-dimensional torus T^2. Then the total homology class of the i-th D-brane is given by

$$\Pi^{(i)} = \prod_{j=1}^{p} \left(n^{(i)}_j \, [\mathbf{2j-1}] + m^{(i)}_j \, [\mathbf{2j}] \right). \tag{2.6}$$

As was first derived in the Type I case in [5] and generalized to Type II in [7], the RR-tadpole cancellation conditions simply mean that the total homology class is zero

$$\sum_{i=1}^{K} N^{(i)} \, \Pi^{(i)} = 0. \tag{2.7}$$

Expanding (2.7) yields 2^p non-trivial conditions for the wrapping numbers $(n^{(i)}_j, m^{(i)}_j)$ [1]. By T-duality the same conditions have to be satisfied for the gauge fluxes (2.5), where now $(n^{(i)}_j, m^{(i)}_j)$ have the interpretation of magnetic charges respectively wrapping numbers on T^2_j.

[1] A consequence of the condition (2.7) is that the homological class is preserved for any marginal deformation of the D-branes. Strictly speaking, this was derived only for the case of D-branes filling all non-compact directions, however even in the general case of Dp-Dp branes the homological class does not change [9]. We are grateful to A. Uranga for pointing out an error in an earlier version of this paper.

Due to the compactification, generically two D-branes at angles have more than one intersection point. In fact the intersection number for two branes with wrapping numbers $(n_j^{(1)}, m_j^{(1)})$ and $(n_j^{(2)}, m_j^{(2)})$ is given by

$$I_{12} = \prod_{j=1}^{p} \left(n_j^{(1)}\, m_j^{(2)} - m_j^{(1)}\, n_j^{(2)} \right). \tag{2.8}$$

Since the massless bi-fundamental chiral fermions are localized at those intersection points, they now appear with an extra multiplicity I_{12}. Due to T-duality the same extra factor must appear for D-branes with background flux, even though in the latter case this factor is not that obvious.

3. Bound states of D$(2p)$-D0 systems

In this section we will discuss the cases of D2-D0, D4-D0, D6-D0 and D8-D0 branes separately and will freely jump between the flux picture and the more intuitive D-branes at angles picture. For the compact case we need some by-standing D-branes to satisfy the tadpole condition (2.7). Nevertheless, for analyzing the decay we can focus on single D$(2p)$-D0 brane pairs.

3.1. D2-D0

As is clear from the D-branes at angles picture two such D1-branes preserve supersymmetry only when they are parallel, i.e. $\Phi_1 = 0$. If they are anti-parallel, $\Phi_1 = \pi$, they describe a D1-$\overline{\text{D}}$1 brane pair. This is also evident from the annulus partition function for open strings stretched between two D1-branes

$$A_{D1,D1} = \int_0^\infty \frac{dt}{t^5} \frac{1}{2} \sum_{\alpha,\beta\in\{0,1/2\}} (-1)^{2(\alpha+\beta)}\, e^{2i\alpha\Phi_1}\, e^{i\pi/2}\, \frac{\vartheta\left[{-\beta \atop \alpha}\right]^3 \vartheta\left[{-\Phi_1/\pi-\beta \atop \alpha}\right]}{\eta^9\, \vartheta\left[{-\Phi_1/\pi-1/2 \atop 1/2}\right]}, \tag{3.1}$$

which only vanishes for $\Phi_1 = 0$. If the two branes are not parallel, a tachyon develops in the NS sector of open strings stretched between the two D-branes. Therefore, the two D-branes will decay into a new configuration of D-branes wrapping the same homological cycle but with less energy. In the case $\Phi_1 \neq 0, \pi$ the decay product is simply the flat D-brane with wrapping numbers $(n_j^{(1)} + n_j^{(2)}, m_j^{(1)} + m_j^{(1)})$. As long as $\Phi_1 \neq 0, \pi$, due to the triangle inequality the resulting brane has smaller volume than the sum of the two volumes of the original D-branes at angles. Moreover, since after the decay one is left with only one flat brane, the configuration preserves maximal supersymmetry and is therefore $\frac{1}{2}$BPS. Note, that in accordance with the Higgs mechanism proposed in [2] only the diagonal $U(1)$ of the former $U(1)\times U(1)$ gauge symmetry survives after condensation of the bi-fundamental tachyon (Higgs-field).

3.2. D4-D0

Computing the NS ground state energy one finds that for

$$\Phi_1 + \Phi_2 = 0 \tag{3.2}$$

the configuration is $\frac{1}{4}$BPS and that for $\Phi_1 + \Phi_2 \neq 0$ one gets a tachyon. To simplify the presentation, in (3.2) we choose the angles in such a way that we have positive signs everywhere. For $\Phi_1 + \Phi_2 = 0$ this tachyon becomes marginal and deforms the two intersecting D-branes with their singular intersection point into a smooth extended 2-cycle preserving the same amount of supersymmetry [9]. In the T-dual picture, this corresponds to deforming a singular gauge bundle (zero size instanton) into a nonsingular gauge bundle (thick instanton) with the same energy.

Therefore, due to Sen's philosophy [11] in the tachyonic case we expect the system to decay into a supersymmetric 2-cycle wrapping the homological cycle

$$\Pi_3 = \Pi_1 + \Pi_2. \tag{3.3}$$

Again, after the decay one ends up with an object which is $\frac{1}{4}$BPS. In contrast to the D2-D0 case, here the stable object can not again be a flat D2-brane, as this would preserve $\frac{1}{2}$ of the type II supersymmetry.

Equivalently, this can be seen from the following purely topological argument: The self-intersection number $\Pi_3 \cdot \Pi_3 = 2\Pi_1 \cdot \Pi_2 > 0$. But a flat D2-brane could be moved off of itself by a shift, therefore would have self-intersection number 0.

In the following we derive the supersymmetric 2-cycle condition simply by lifting the global condition $\Phi_1 + \Phi_2 = 0$ to a local one. Remember that the general characterization of a supersymmetric p-cycle is that it is a special Lagrangian submanifold, which means that for an embedding map $i : (p - \text{cycle}) \longrightarrow \mathbb{C}^p$ the two conditions

$$\begin{aligned} i^* \,\mathrm{Im}\,\Omega &= 0 \\ i^*\, \omega &= 0 \end{aligned} \tag{3.4}$$

are satisfied [12]. In (3.4) $\Omega = dz_1 \wedge \cdots \wedge dz_p$ denotes the holomorphic volume form on $\mathbb{C}^p$ and $\omega = \frac{1}{2i} \sum dz_i \wedge d\bar{z}_i$ the standard Kähler form. Instead of starting with the conditions (3.4) and derive the explicit form of the first order partial differential equation as was done in [10], we start with the condition

$$\Phi_1 + \Phi_2 = 0 \Longleftrightarrow \cot\Phi_1 + \cot\Phi_2 = \frac{\partial x_1}{\partial y_1} + \frac{\partial x_2}{\partial y_2} = 0, \tag{3.5}$$

where so far x_j depends only on y_j for the same index j. A cycle in general position can be descibed by the graph

$$x_1 = x_1(y_1, y_2), \qquad x_2 = x_2(y_1, y_2). \tag{3.6}$$

To derive the generalization of (3.5) for this case note that under T-duality the partial derivatives transform into the field strength of the gauge field. There we can apply the most general rotation such that we can T-dualize back.

So put the partial derivatives into an antisymmetric matrix and apply rotations such that each 2×2-block remains off-diagonal:

$$\begin{pmatrix} 0 & -\frac{\partial \tilde{x}_1}{\partial \tilde{y}_1} & 0 & -\frac{\partial \tilde{x}_1}{\partial \tilde{y}_2} \\ \frac{\partial \tilde{x}_1}{\partial \tilde{y}_1} & 0 & \frac{\partial \tilde{x}_2}{\partial \tilde{y}_1} & 0 \\ 0 & -\frac{\partial \tilde{x}_2}{\partial \tilde{y}_1} & 0 & -\frac{\partial \tilde{x}_2}{\partial \tilde{y}_2} \\ \frac{\partial \tilde{x}_1}{\partial \tilde{y}_2} & 0 & \frac{\partial \tilde{x}_2}{\partial \tilde{y}_2} & 0 \end{pmatrix} = R \begin{pmatrix} 0 & -\frac{\partial x_1}{\partial y_1} & 0 & 0 \\ \frac{\partial x_1}{\partial y_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial x_2}{\partial y_2} \\ 0 & 0 & \frac{\partial x_2}{\partial y_2} & 0 \end{pmatrix} R^{-1} \tag{3.7}$$

with

$$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \otimes \mathbb{1}_2 = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{3.8}$$

Inspection of the resulting matrix on the left hand side of (3.7) yields the general conditions for a supersymmetric 2-cycle (removing the tilde's)

$$\begin{aligned} \frac{\partial x_1}{\partial y_2} &= \frac{\partial x_2}{\partial y_1} \\ \frac{\partial x_1}{\partial y_1} &= -\frac{\partial x_2}{\partial y_2}. \end{aligned} \tag{3.9}$$

The first follows from the special form of the rotation matrix R whereas the latter is (3.5). For the case of D4-branes, this condition is invariant under the rotation (3.8). We will see below that the corresponding equation for larger p will not be invariant but we will have to find its "invariantization". A counting of parameters (a general antisymmetric 4×4-matrix has six independant entries, two of them have to vanish due to the ansatz (3.6), there was one independent entry in block-diagonal form, and we have two equations and one angle) shows that these two equations are sufficient to characterize the general rotated matrix (3.7) that obeys supersymmetry.

The equations (3.9) are precisely the Cauchy-Riemann differential equations for an anti-holomorphic map. Thus, by these elementary steps we have recovered the well known

result, that supersymmetric 2-cycles are (anti-)holomorphic curves. Applying T-duality along the x_j-directions maps coordinates to gauge fields

$$x_j \longrightarrow A_{(2j-1)}. \tag{3.10}$$

As a consequence, the matrix (3.7) is mapped to

$$F = \begin{pmatrix} 0 & F_{12} & 0 & F_{14} \\ -F_{12} & 0 & F_{23} & 0 \\ 0 & -F_{23} & 0 & F_{34} \\ -F_{14} & 0 & -F_{34} & 0 \end{pmatrix} \tag{3.11}$$

and the relations (3.9) are mapped to the anti-self-duality constraint

$$F = -(*F). \tag{3.12}$$

Thus, we realize that holomorphicity of a complex curve and self-duality of a gauge field are related by T-duality. Note, that by this T-duality we can not obtain the most general form of the field strength, as some of the components are necessarily zero. So far, for the 2-cycle we have not learned anything new, but the same method can also be applied to derive the corresponding 3-cycle and 4-cycle conditions and their T-dual versions.

One might wonder how we could extract a local condition from the global condition (3.3) for flat branes. But (3.4) is an algebraic condition on the tangent space of the cycle. Thus, only first derivatives are involved (as we expect for BPS states) so that equations (3.9) provide us with an equivalent characterization of a supersymmetric 2-cycle.

3.3. D6-D0

Computing the NS ground state energy one finds that for

$$\Phi_1 + \Phi_2 + \Phi_3 = \pi \tag{3.13}$$

the configuration is $\frac{1}{8}$BPS and that for $\Phi_1 + \Phi_2 + \Phi_3 < \pi$ one gets a tachyon in the NS sector. On the other side of the supersymmetry locus, $\Phi_1 + \Phi_2 + \Phi_3 > \pi$, the system is tachyon-free and therefore stable.

This nicely complements the geometric picture that one has for the intersection of two special Lagrangian planes in $\mathbb{C}^3$ (see [13]): Precisely if $\Phi_1 + \Phi_2 + \Phi_3 = \pi$ the two planes are special Lagrangian, and only if $\Phi_1 + \Phi_2 + \Phi_3 < \pi$ one can deform the union of the two planes towards lower volume.

Again, for $\Phi_1+\Phi_2+\Phi_3=\pi$ the tachyonic mode becomes marginal and we expect that it deforms the two intersecting D-branes into a smooth 3-cycle preserving the same amount of supersymmetry. In the T-dual picture this will correspond to a smoothing of the gauge bundle.

In the non-compact case of two 3-planes in $\mathbb{C}^3$ one can even explicitly write down such a deformation. To simplify the equations take $\Phi_1=\Phi_2=\Phi_3=\frac{\pi}{3}$, then (one connected component of)

$$K_c=\left\{(zx_1,zx_2,zx_3)\in\mathbb{C}^3:(x_1,x_2,x_3)\in S^2,\operatorname{Im} z^3=c\right\} \tag{3.14}$$

is special Lagrangian and asymptotically approaches the two planes. The intersection with the complex plane $(\mathbb{C},0,0)$ is depicted in figure 2 for different c.

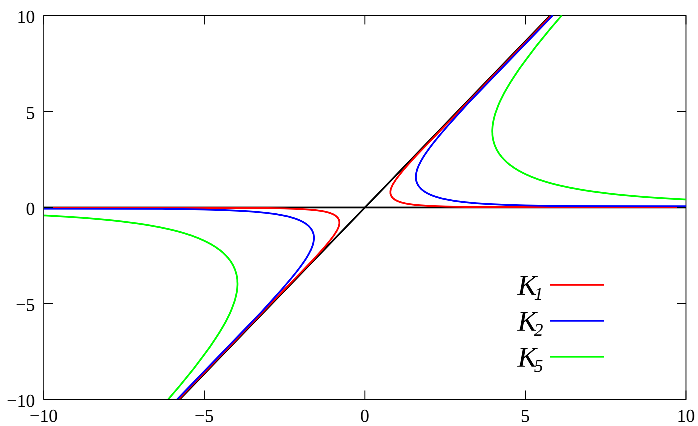

Fig.2: Deformation into a smooth special Lagrangian submanifold

As in the D4-D0 case, we expect that tachyon condensation leads to a $\frac{1}{8}$BPS bound state, which can be described as a necessarily non-flat supersymmetric 3-cycle wrapping around the homological cycle $\Pi_3=\Pi_1+\Pi_2$. To determine the general equation satisfied by such cycles we again require that the angle conditions are satisfied locally

$$\cot\left(\Phi_1+\Phi_2+\Phi_3\right)=\frac{\cot\Phi_1\cot\Phi_2\cot\Phi_3-\cot\Phi_1-\cot\Phi_2-\cot\Phi_3}{\cot\Phi_1\cot\Phi_2+\cot\Phi_2\cot\Phi_3+\cot\Phi_3\cot\Phi_1-1}=-\infty \tag{3.15}$$

leading to

$$\frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2}+\frac{\partial x_1}{\partial y_1}\frac{\partial x_3}{\partial y_3}+\frac{\partial x_2}{\partial y_2}\frac{\partial x_3}{\partial y_3}=1, \tag{3.16}$$

as long as x_j depends only on y_j. The general 3-cycle conditions for a graph

$$x_1=x_1(y_1,y_2,y_3),\qquad x_2=x_2(y_1,y_2,y_3),\qquad x_3=x_3(y_1,y_2,y_3) \tag{3.17}$$

can be obtained by applying the most general $SO(3)$ rotation and reading off the relations for the rotated coordinates.

We obtain

$$\frac{\partial x_2}{\partial y_1} = \frac{\partial x_1}{\partial y_2}, \quad \frac{\partial x_3}{\partial y_1} = \frac{\partial x_1}{\partial y_3}, \quad \frac{\partial x_3}{\partial y_2} = \frac{\partial x_2}{\partial y_3},$$
$$\frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2} + \frac{\partial x_1}{\partial y_1}\frac{\partial x_3}{\partial y_3} + \frac{\partial x_2}{\partial y_2}\frac{\partial x_3}{\partial y_3} - \frac{\partial x_2}{\partial y_1}\frac{\partial x_1}{\partial y_2} - \frac{\partial x_3}{\partial y_1}\frac{\partial x_1}{\partial y_3} - \frac{\partial x_3}{\partial y_2}\frac{\partial x_2}{\partial y_3} = 1. \tag{3.18}$$

These four conditions agree completely with the general result obtained in [10] restricted to a graph (3.17). Employing T-duality we can now easily derive the generalization of the self-duality constraint (3.12) to $\frac{1}{8}$BPS gauge configurations. For a restricted gauge field of the form

$$F = \begin{pmatrix} 0 & F_{12} & 0 & F_{14} & 0 & F_{16} \\ -F_{12} & 0 & F_{23} & 0 & F_{25} & 0 \\ 0 & -F_{23} & 0 & F_{34} & 0 & F_{36} \\ -F_{14} & 0 & -F_{34} & 0 & F_{45} & 0 \\ 0 & -F_{25} & 0 & -F_{45} & 0 & F_{56} \\ -F_{16} & 0 & -F_{36} & 0 & -F_{56} & 0 \end{pmatrix} \tag{3.19}$$

the conditions for preserving $\frac{1}{8}$ of the 32 supercharges are

$$F_{14} = -F_{23}, \quad F_{16} = -F_{25}, \quad F_{36} = -F_{45}$$
$$1 = F_{12}\,F_{34} + F_{12}\,F_{56} + F_{34}\,F_{56} + F_{14}\,F_{23} + F_{16}\,F_{25} + F_{36}\,F_{45}. \tag{3.20}$$

As we mentioned before, the form of the constraint for the most general choice of the gauge field strength can not be derived by employing T-duality. Different $\frac{1}{8}$BPS gauge configurations were discussed in [14].

3.4. D8-D0

In this case the configuration is $\frac{1}{16}$BPS if

$$\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 = \pi\epsilon \tag{3.21}$$

with the disjoint branches $\epsilon = 1, 2$. For $\epsilon = 2$ the system has neither massless nor tachyonic states and will be stable. Note, that the usual supersymmetric D8-D0 system corresponds to $\Phi_j = \pi/2$ for all j. For the other branch $\epsilon = 1$ the situation is very similar to the D6-D0 case. For $\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 > \pi$ the system is stable whereas for $\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 < \pi$ it develops a tachyon. Again, we expect that after tachyon condensation the system will decay into a supersymmetric 4-cycle in $\mathbb{C}^4$ and the equations governing the 4-cycle

$$\frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2}\frac{\partial x_3}{\partial y_3} + \frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2}\frac{\partial x_4}{\partial y_4} + \frac{\partial x_1}{\partial y_1}\frac{\partial x_3}{\partial y_3}\frac{\partial x_4}{\partial y_4} + \frac{\partial x_2}{\partial y_2}\frac{\partial x_3}{\partial y_3}\frac{\partial x_4}{\partial y_4} -$$
$$\frac{\partial x_1}{\partial y_1} - \frac{\partial x_2}{\partial y_2} - \frac{\partial x_3}{\partial y_3} - \frac{\partial x_1}{\partial y_1} = 0. \tag{3.22}$$

can be derived from the corresponding local angle relation. The generalization for a general graph

$$x_1 = x_1(y_1, y_2, y_3, y_4), \qquad x_2 = x_2(y_1, y_2, y_3, y_4),$$
$$x_3 = x_3(y_1, y_2, y_3, y_4), \qquad x_4 = x_4(y_1, y_2, y_3, y_4) \tag{3.23}$$

can be found by applying the most general $SO(4)$ rotation and extracting the conditions

$$\frac{\partial x_2}{\partial y_1} = \frac{\partial x_1}{\partial y_2}, \quad \frac{\partial x_3}{\partial y_1} = \frac{\partial x_1}{\partial y_3}, \quad \frac{\partial x_4}{\partial y_1} = \frac{\partial x_1}{\partial y_4}, \quad \frac{\partial x_3}{\partial y_2} = \frac{\partial x_2}{\partial y_3}, \quad \frac{\partial x_4}{\partial y_2} = \frac{\partial x_2}{\partial y_4}, \quad \frac{\partial x_4}{\partial y_3} = \frac{\partial x_3}{\partial y_4},$$

$$\begin{aligned}
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&\frac{\partial x_3}{\partial y_1}\frac{\partial x_4}{\partial y_1}\frac{\partial x_4}{\partial y_3} + \frac{\partial x_3}{\partial y_2}\frac{\partial x_4}{\partial y_3}\frac{\partial x_3}{\partial y_4} - \frac{\partial x_2}{\partial y_1}\frac{\partial x_1}{\partial y_2}\frac{\partial x_4}{\partial y_4} - \frac{\partial x_3}{\partial y_1}\frac{\partial x_1}{\partial y_3}\frac{\partial x_4}{\partial y_4} - \frac{\partial x_3}{\partial y_2}\frac{\partial x_2}{\partial y_3}\frac{\partial x_4}{\partial y_4} - \\
&\frac{\partial x_2}{\partial y_1}\frac{\partial x_1}{\partial y_2}\frac{\partial x_3}{\partial y_3} - \frac{\partial x_4}{\partial y_1}\frac{\partial x_1}{\partial y_4}\frac{\partial x_3}{\partial y_3} - \frac{\partial x_4}{\partial y_2}\frac{\partial x_2}{\partial y_4}\frac{\partial x_3}{\partial y_3} - \frac{\partial x_3}{\partial y_1}\frac{\partial x_1}{\partial y_3}\frac{\partial x_2}{\partial y_2} - \frac{\partial x_4}{\partial y_1}\frac{\partial x_1}{\partial y_4}\frac{\partial x_2}{\partial y_2} - \\
&\frac{\partial x_4}{\partial y_3}\frac{\partial x_3}{\partial y_4}\frac{\partial x_2}{\partial y_2} - \frac{\partial x_3}{\partial y_2}\frac{\partial x_2}{\partial y_3}\frac{\partial x_1}{\partial y_1} - \frac{\partial x_4}{\partial y_2}\frac{\partial x_2}{\partial y_4}\frac{\partial x_1}{\partial y_1} - \frac{\partial x_4}{\partial y_3}\frac{\partial x_3}{\partial y_4}\frac{\partial x_1}{\partial y_1} - \\
&\frac{\partial x_1}{\partial y_1} - \frac{\partial x_2}{\partial y_2} - \frac{\partial x_3}{\partial y_3} - \frac{\partial x_1}{\partial y_1} = 0.
\end{aligned} \tag{3.24}$$

In deriving these increasingly complicated expressions it proved to be helpful to use that in the limit $\frac{\partial x_j}{\partial y_j} \to \infty$ one gets back the 3-cycle conditions. The T-dual conditions for a $\frac{1}{16}$BPS gauge configuration of the form

$$F = \begin{pmatrix}
0 & F_{12} & 0 & F_{14} & 0 & F_{16} & 0 & F_{18} \\
-F_{12} & 0 & F_{23} & 0 & F_{25} & 0 & F_{27} & 0 \\
0 & -F_{23} & 0 & F_{34} & 0 & F_{36} & 0 & F_{38} \\
-F_{14} & 0 & -F_{34} & 0 & F_{45} & 0 & F_{47} & 0 \\
0 & -F_{25} & 0 & -F_{45} & 0 & F_{56} & 0 & F_{58} \\
-F_{16} & 0 & -F_{36} & 0 & -F_{56} & 0 & F_{67} & 0 \\
0 & -F_{27} & 0 & -F_{47} & 0 & -F_{67} & 0 & F_{78} \\
-F_{18} & 0 & -F_{38} & 0 & -F_{58} & 0 & -F_{78} & 0
\end{pmatrix} \tag{3.25}$$

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$$F_{14} = -F_{23}, \quad F_{16} = -F_{25}, \quad F_{18} = -F_{27}, \quad F_{36} = -F_{45}, \quad F_{38} = -F_{47}, \quad F_{58} = -F_{67},$$

$$\begin{aligned}
&F_{12}\,F_{34}\,F_{56} + F_{12}\,F_{34}\,F_{78} + F_{12}\,F_{56}\,F_{78} + F_{34}\,F_{56}\,F_{78} + F_{14}\,F_{16}\,F_{36} + \\
&F_{14}\,F_{18}\,F_{38} + F_{16}\,F_{18}\,F_{58} + F_{36}\,F_{38}\,F_{58} - F_{23}\,F_{25}\,F_{45} - F_{23}\,F_{27}\,F_{47} - \\
&F_{25}\,F_{27}\,F_{67} - F_{45}\,F_{47}\,F_{67} + F_{14}\,F_{23}\,F_{78} + F_{16}\,F_{25}\,F_{78} + F_{36}\,F_{45}\,F_{78} + \\
&F_{14}\,F_{23}\,F_{56} + F_{18}\,F_{27}\,F_{56} + F_{38}\,F_{47}\,F_{56} + F_{16}\,F_{25}\,F_{34} + F_{18}\,F_{27}\,F_{34} + \\
&F_{58}\,F_{67}\,F_{34} + F_{36}\,F_{45}\,F_{12} + F_{38}\,F_{47}\,F_{12} - F_{58}\,F_{67}\,F_{12} - \\
&F_{12} - F_{34} - F_{56} - F_{78} = 0.
\end{aligned} \tag{3.26}$$

Thus, we have seen that a supersymmetric p-cycle in $\mathbb{C}^p$ can be characterized as an object which locally satisfies the familiar angle relation $\sum \Phi_j = \pi$. Solutions to the resulting conditions for 3-cycles and 4-cycles are not known so far, but might have some impact on our understanding of non-perturbative aspects of $\mathcal{N} = 1$ gauge theories along the lines of [15,10]. It would also be interesting to find the general form of the supersymmetry conditions for the gauge field strength.

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